Quarterly Progress Report

FAILURE CRITERIA
FOR VISCOELASTIC MATERIALS

W. G. Knauss

GALCIT SM 68-19

DECEMBER 1968

This work was supported by the
National Aeronautics and Space Administration
Research Grant No. NGL-05-002-005
GALCIT 120

Firestone Flight Sciences Laboratory
Graduate Aeronautical Laboratories
California Institute of Technology
Pasadena, California

FRACTURE MECHANICS IN LINEARLY VISCOELASTIC MATERIALS

During the last three months our work was directed at solidification of the results on crack propagation in linearly viscoelastic materials.

One problem of particular concern was an apparent discrepancy in results based, on the one hand, on allowing the crack to propagate in infinitesimal jumps, and allowing it to grow continuously, on the other. This problem has been resolved in the following way:

When the material at the tip of the crack unloads as the crack propagates into it, the work done by the unloading forces against the receding crack boundary depends on the precise distribution of the stresses at the crack tip. Since the region about the crack tip is under high strain nonlinear material properties are invoked and stress analysis becomes impossible. At this point, one has to make a reasonable assumption concerning the stress history to which the material is subjected. Variation in the final differential equation describing the motion of the crack are the result of making an assumption regarding the stress history. For instance, two different assumptions have lead to results for a crack propagating in viscoelastic strip under constant strain ϵ with velocity $\dot{c}$

$$\epsilon^2 D^*_{cr}\left(\frac{\Delta a^*}{\dot{c}}\right) = \text{const} \tag{1}$$

$$\epsilon^2 D_{cr}\left(\frac{\Delta a}{\dot{c}}\right) = \text{const} \tag{2}$$

where $D^*_{cr}(t) = \frac{1}{t}\int_0^t D_{cr}(t)\, dt$ is the average creep compliance. Now it is interesting to note that $D^*_{cr}(t)$ is a function very similar to $D_{cr}(t)$ with the same limits at $t = 0, \infty$, except that it is shifted, on a logarithmic-time plot, to larger values of time. This shift would only affect the experimental constant $\Delta a(\Delta a^*)$. Comparison of (1) and (2) with experiment yields no discernable differences.

We conclude therefore that although differences in theoretical formulation of the crack propagation problem arise, their practical consequences are apparently not important.

Conversely we conclude that it will be difficult to resolve the question of the crack tip stress distribution and history from crack propagation data, because the latter is not a sensitive function of the former.

This work is presently in the stage of manuscript typing.

As a second concerted effort we have worked on extending our past work to more general loading histories, with the special goal being cyclic loading and its relation to fatigue. To date we have obtained the following results:

The formerly differential equation governing crack growth under steady loading becomes, for time varying loading, an integro-differential equation. This integro-differential equation must be solved for the general case, in particular when the crack grows slowly, such as may be the case in very-long-time fracture. It is natural and desirable from a practical viewpoint to inquire into simplifications and to explore their range of applicability. One such simplification is to assume that the stress rante $\dot{\sigma}$ does not change much while the crack grows a distance Δa(Δa is on the order of angstroms). For this case, which may actually cover a very large range of practical problems, the integro-differential equation reduces to a nonlinear differential equation for the crack length $c(t)$ in an infinite sheet under tension $\sigma(t)$ of the form

$$\left\{\sigma^2(t)\, D\left(\frac{\Delta a}{\dot{c}}\right) + \frac{1}{2}\frac{d\sigma^2(t)}{dt}\,\frac{\Delta a}{\dot{c}}\, D^*\left(\frac{\Delta a}{\dot{c}}\right)\right\} c(t) = \frac{2\Gamma}{\pi} \qquad (3)$$

where $\Gamma =$ const, surface energy

$$D^*(t) = \frac{1}{t}\int_o^t D(\tau)\, d\tau$$

In general equation (3) must be integrated numerically. We are currently investigating an analytical way of integration for sinusoidal variations of the stress

$$\sigma(t) = \sigma_o(1 + \epsilon \sin \omega t)$$

in the hope of arriving at a general parametric solution in ϵ and ω.

In order to compare these calculations with experiment we are reducing experimental data accumulated earlier. It appears that the theory agress so well with experiments that it is necessary to take into account mall (10 per cent) changes in modulus values of Solithane sheets produced in different batches.

THERMO-VISCOELASTIC PROBLEM

One of the nearly unsolved problems of solid rocket technology is that of transient thermo-viscoelasticity. The attached note (GALCIT SM 68-17) illustrates one basic problem, which deals both with the inadequacy of analytical methods as well as experimental verification of a fundamental assumptions, namely that of thermo-rheological simplicity under transient temperature variations.

To contribute to the solution of this problem we have conducted tests on Solithane 113 to measure its heat diffusivity. We found that the diffusivity is not a constant but appears to be an operator similar to a viscoelastic compliance operator, except that the difference between the classical result of constant diffusivity and the operator response is less than for the corresponding problem on compliance measurements. Figure 1 shows measurements of the temperature at the center of a cylinder and comparison with the calculations for a constant diffusivity. Although the deviation is not excessive in terms of the usual variation of the relaxation modulus it should be noted that these small temperature variations may lead to sizeable variations of stresses in viscoelastic materials under time-varying temperature fields because of their extreme temperature sensitivity near the glass transition temperature.

We are currently conducting experiments which allow us to check transient thermo-viscoelastic stress analysis in a rod under a step-temperature drop with theory (see attached note). To this end we have constructed a temperature conditioning device which allows us to flush the surface of a 1/2" diameter sample in a half second with a $-15^{\circ}C$ liquid. This assures a high rate of cooling and deviations from assumed classical behavior should show up most markedly.

It is interesting to note that current transient thermoviscoelastic analysis for solid rocket motors are being conducted on the assumption that the temperature distribution is governed by the classical Fourier heat conduction equation with a constant diffusivity. The current investigation may point out to what extent such calculations may be in error, and that caution is in order before spending large sums on developing thermo-viscoelastic computer algorithms.

DISTRIBUTION LIST

R. Ziem

G. Lewis

M. L. Williams

R. A. Schapery

K. Pister

J. Bollard

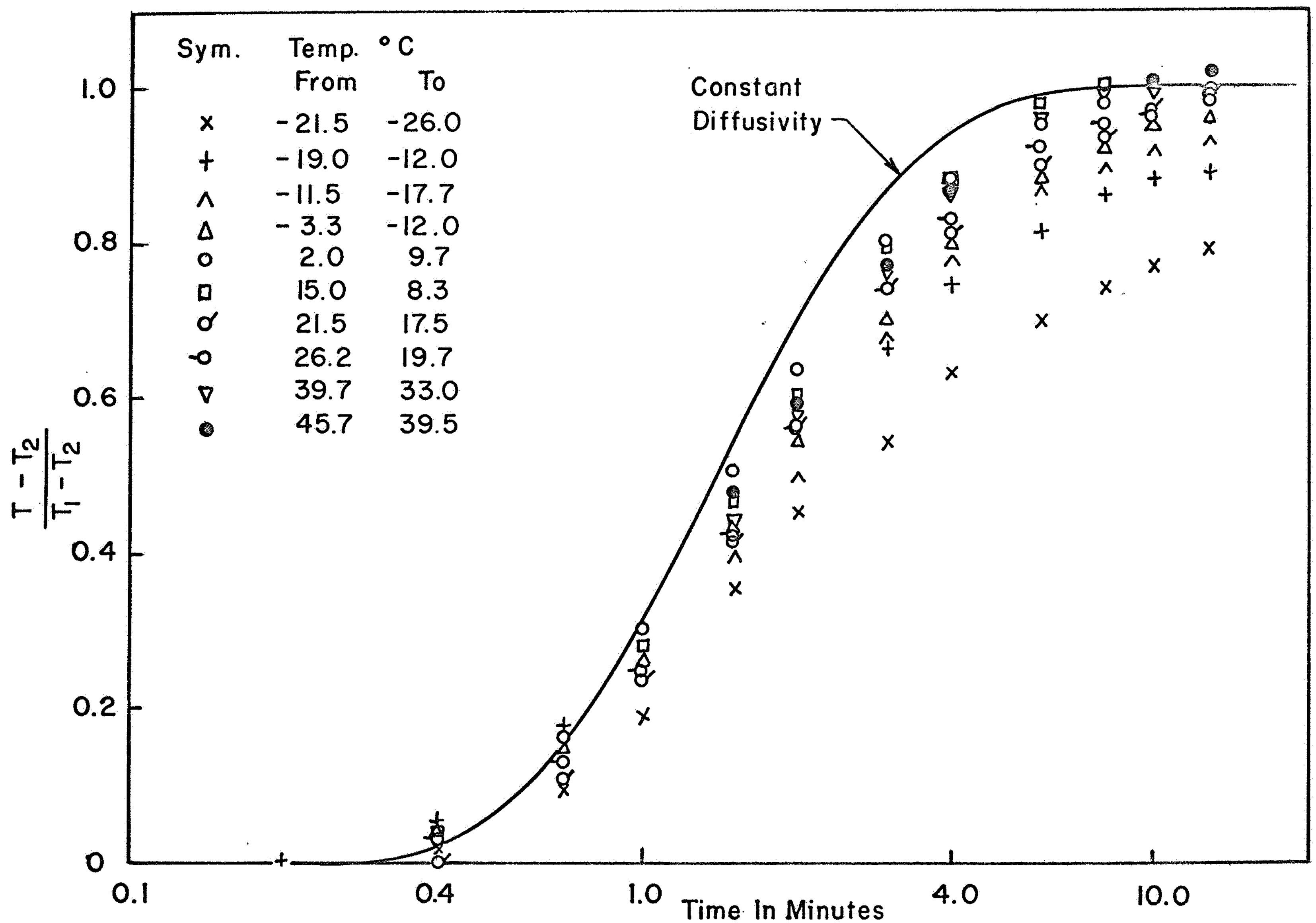

FIG. I TEMPERATURE HISTORY AT ROD CENTER DUE TO A TEMPERATURE STEP ON THE SURFACE